Statistics

Lecture 11

Feb 19-8:47 AM

Critical Values:

SG 21 & SG 22

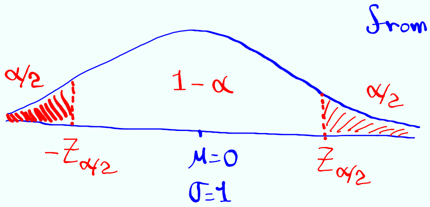
$Z_{\alpha/2}$ → Area on the right Tail

Alpha

$0 < \alpha < 1$

α is the Significance level

$Z_{\alpha/2}$ is a numerical Value that Separates the right area of $\alpha/2$ from the rest.



Middle Area $1-\alpha$

$(1-\alpha)\cdot 100\%$ Confidence level

How to find $Z_{\alpha/2}$

$\boxed{2nd}$ $\boxed{VARS}$

invNorm(, 0, 1)

Left Area, μ, σ

Always round to 3-decimal places.

Nov 10-5:08 PM

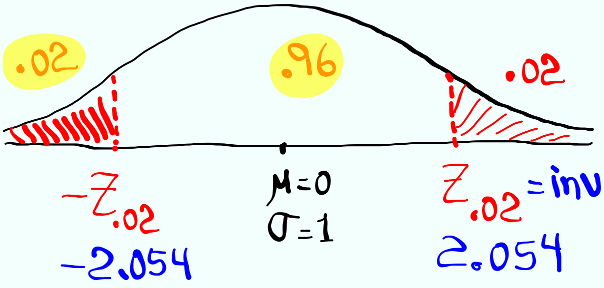
find $Z_{\boxed{.02}}$

$\to$ Right Area

$\to \alpha/2 = .02 \to \alpha = .04$ Significance level

$1 - \alpha = .96$ Middle Area

$.96 = 96\%$ Confidence level



$Z_{.02}$ = invNorm(.98, 0, 1) = $\boxed{2.054}$

Nov 10-5:15 PM

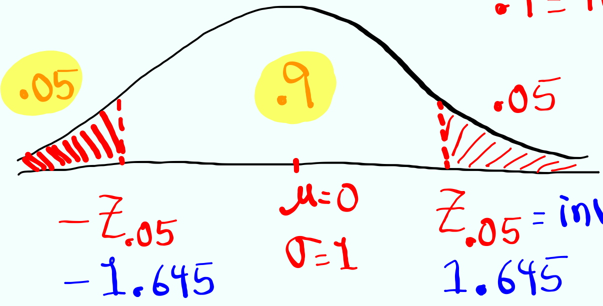
find $\pm Z_{\alpha/2}$ for $\alpha = .1$.

← Sign. level

$\alpha/2 = .05$ area of each tail

$1 - \alpha = .9$ Middle Area

$.9 = 90\%$ $\underbrace{\text{Conf. level}}_{\text{C-level}}$



$Z_{.05}$ = invNorm(.95, 0, 1) = $\boxed{1.645}$

Nov 10-5:20 PM

find $\pm Z_{\alpha/2}$ for 99% $\underbrace{\text{Confidence level.}}_{\text{C-level}}$

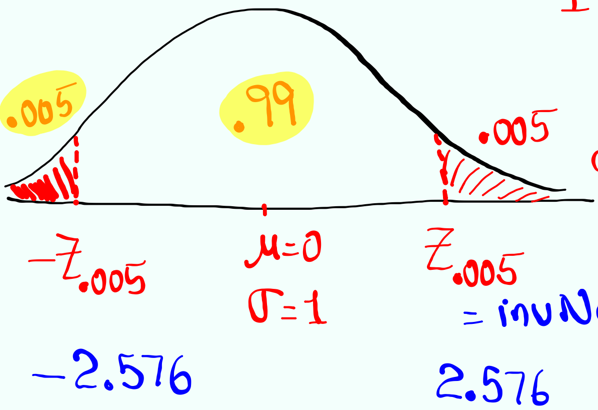
$\underbrace{\qquad\qquad\qquad}$

Middle Area = .99

$1-\alpha=.99$

$\alpha=.01$ Sig. level

$\alpha/2=.005$ Area of each tail



$= \text{invNorm}(.995, 0, 1) = \boxed{2.576}$

Nov 10-5:25 PM

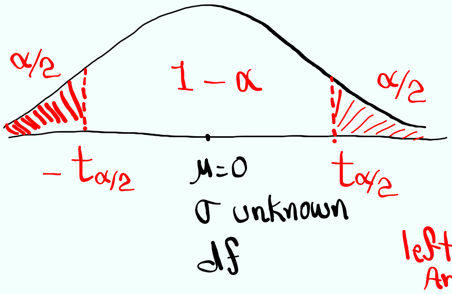
$t_{\alpha/2}$ t-Dist.

1) Symmetric, Bell-Shape, Total Area=1

2) Mean = Mode = Median

3) $\mu=0$, σ unknown

4) It comes with $\underbrace{\text{degrees of freedom}}_{df}$



How to find $t_{\alpha/2}$

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$

$\text{invT}(\underbrace{\qquad}_{\text{left Area}}, \underline{\quad}\,df)$

Nov 10-5:30 PM

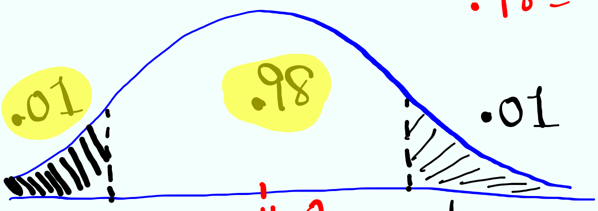
find $t_{.01}$ with df = 9.

$\to$ Right Tail Area

$\to \alpha/2 = .01 \to \alpha = .02$ Sig. level.

$1 - \alpha = .98$ Middle Area

.98 = 98% C-level



$-t_{.01}$ $\quad \mu = 0$ $\quad t_{.01} = \text{invT}(.99, 9) = \boxed{2.821}$

-2.821 $\quad \sigma$ unknown, df = 9 $\quad 2.821$

Nov 10-5:35 PM

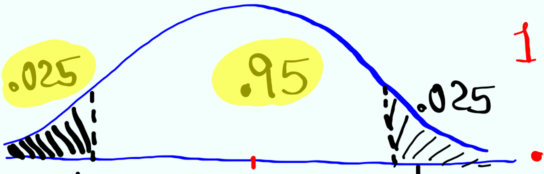
find $\pm t_{\alpha/2}$ for $\alpha = .05$ and df = 24.

Sig. level

$\alpha/2 = .025$ Area of each tail

$1 - \alpha = .95$ Middle Area

.95 = 95% C-level



$-t_{.025}$ $\quad \mu = 0$ $\quad t_{.025}$

-2.064 $\quad \sigma$ unknown, df = 24 $\quad 2.064$

$= \text{invT}(.975, 24) = \boxed{2.064}$

Nov 10-5:40 PM

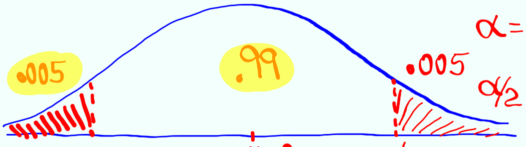
find $\pm t_{\alpha/2}$ for $\underbrace{99\% \text{ C-level}}$ with $df = 99$.

Middle Area $= .99$

$1 - \alpha = .99$

$\alpha = .01$ Sig. level

$\alpha/2 = .005$ Area of each tail



$-t_{.005}$ $\quad \mu = 0$ $\quad t_{.005} = \text{invT}(.995, 99) = \boxed{2.626}$

σ unknown

-2.626 $\quad df = 99$ $\quad 2.626$

As df increases,

$$t_{\alpha/2} \approx z_{\alpha/2}$$

Nov 10-5:44 PM

degrees of freedom

It will be given to us or can be determined by the topic.

20 students, I bring 20 donuts.

First student → 20 choices

Second " → 19 choices

Third " → 18 "

⋮

Last student → 0 choices (1 donut left)

$\boxed{df = 19}$

7 clean shirt

wear one clean shirt daily

Monday 7 choices

Tuesday 6 choices

Wednesday 5 " → … Sunday 0 choices (1 clean shirt)

$\boxed{df = 6}$

Nov 10-5:50 PM

Estimating Parameters

SG 21

To estimate Population		we use Sample
Proportion	P	Proportion $\hat{p}$
Mean	μ	Mean $\bar{x}$
Standard Deviation	σ	Standard Deviation s

Point-estimate

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estimation is a range-of-Values which is called Confidence Interval.

Every estimation comes with Confidence level $(1-\alpha)\cdot 100\%$

If C-level is not given, we use 95%.

Nov 10-5:56 PM

Estimating Population Proportion P

Answer Format $\quad < P <$

$$\hat{p} - E < P < \hat{p} + E$$

Sample Proportion

Point-estimate

Margin of error

$$\hat{p} = \frac{x}{n}$$

x ← # of favorable responces

n ← Sample Size

$$\hat{q} = 1 - \hat{p}$$

$$E = Z_{\alpha/2}\cdot\sqrt{\frac{\hat{p}\,\hat{q}}{n}}$$

Critical Value for $(1-\alpha)\cdot 100\%$ C-level.

Nov 10-6:04 PM

I Surveyed 100 students and 80 of them had iPhone.

$n = 100$ $x = 80$

$\hat{p} = \frac{x}{n} = \frac{80}{100} = .8$

C-level: .9 $\hat{q} = 1 - \hat{p} = .2$

Find 90% Conf. interval for the Proportion of all Students that have iPhone.

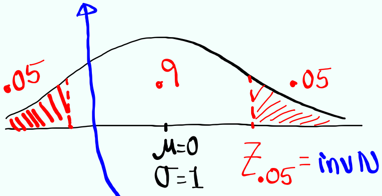
$\hat{p} - E < p < \hat{p} + E$

$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$.8 - .07 < P < .8 + .07$

$= 1.645 \cdot \sqrt{\frac{(.8)(.2)}{100}} \approx \boxed{.07}$

$\boxed{.73 < p < .87}$



we are 90% Confident that between 73% & 87% of all students have iPhone.

$Z_{.05} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

Nov 10-6:09 PM

Using TI:

STAT → TESTS ↓ 1-PropZInt

$\boxed{.73 < P < .87}$

$x = 80$

$n = 100$

C-level = .9

Calculate

$E = \frac{.87 - .73}{2} = \boxed{.07}$

$\hat{p} = \frac{.87 + .73}{2} = \boxed{.8}$

Nov 10-6:19 PM

I Surveyed 150 Students and 20% of them were taking buses to come to School.

$n=150$

$\hat{p}=.2$

$x = n\hat{p} = 150(.2) = \boxed{30}$

if decimal → Round-up

Find 98% Confidence Interval for the Prop. of all Students that take buses to Come to School.

$\boxed{.12 < P < .28}$

C-level: .98

1-Prop Z Int

$x = 30$

$n = 150$

C-level = .98

$$E = \frac{.28 - .12}{2} = \boxed{.08}$$

$$\hat{p} = \frac{.28 + .12}{2} = \boxed{.2}$$

We are 98% Confident that between 12% & 28% of all Students take buses to Come to School.

Nov 10-6:25 PM

I Surveyed 245 students, and 8.2% of them were left-handed.

$n=245$ $\hat{p}=.082$

$x = n\hat{p} = 245(.082) = 20.09 \approx \boxed{21}$

if decimal → Round-up

NO, C-level → .95

Find Confidence interval for the Prop. of all Students that are left-handed.

1-Prop Z Int

$x:21$

$n=245$

C-level: .95

Calculate

$\boxed{.05 < P < .12}$

We are 95% Confident that between 5% & 12% of all Students are left-handed.

$$E = \frac{.12 - .05}{2} = 0.035 \approx \boxed{.04}$$

$$\hat{p} = \frac{.12 + .05}{2} = \boxed{0.085} \approx \boxed{8.5\%}$$

Nov 10-6:34 PM

How do we determine the minimum Sample Size: n

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$$

we use algebra to Solve for n

$$\boxed{n = \hat{p}\,\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2}$$

if decimal, Always round-up

If $\hat{p}$ & $\hat{q}$ are unknown, use .5 for each,

$$\boxed{n = .25\left(\frac{z_{\alpha/2}}{E}\right)^2}$$

Nov 10-6:43 PM

Find minimum Sample Size needed to

c-level: .98

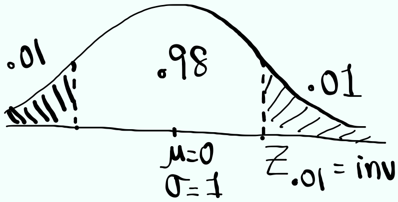
Construct 98% Conf. interval for pop. Prop.

if $\hat{p} = .7$ and $E = .05$.

$$n = \hat{p}\,\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2 = (.7)(.3)\left(\frac{2.326}{.05}\right)^2$$

$$= 454.463$$

$$\approx \boxed{455}$$



$$z_{.01} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$$

Suppose $\hat{p}$ & $\hat{q}$ were unknown

$$n = .25\left(\frac{2.326}{.05}\right)^2 = 541.0276 \approx \boxed{542}$$

Nov 10-6:48 PM

Find min. Sample Size needed to construct conf. interval for pop. prop. with $\hat{p}=.6$ and error not to exceed 10%.

$n=?$ $\hat{p}=.6$ $E=10\%=.1$

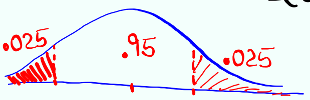
no C-level $\hat{q}=.4$

C-level: .95

$$n=\hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

$$=(.6)(.4)\left(\frac{1.960}{.1}\right)^2$$

$$=92.1984\approx\boxed{93}$$



$Z_{.025}$=invNorm(.975,0,1)=$\boxed{1.960}$

Suppose we want E=.05

$$n=(.6)(.4)\left(\frac{1.960}{.05}\right)^2=368.7936\approx\boxed{369}$$

Nov 10-6:55 PM